Trapped Ions and Beam Coherent Instability

P. Zhou, P. L. Colestock and S. J. Werkema
Fermi National Accelerator Laboratory*
P.O. Box 500, Batavia, IL 60510

Abstract

In accelerators with negatively charged beams, ions generated from the residual gas molecules may be trapped by the beam. Trapped ions may interact resonantly with the beam and cause a beam-ion coherent instability. This coherent instability bears many similarities to the resistive wall instability and can present important limitations to those machines' operation. A description of this effect requires a treatment of the beam coherent instability including both the normal machine wake field and the interaction with ions. We present a linear approach incorporating contributions from the machine impedance as well as ion forces. It also includes spreads in beam and ion frequencies and thus Landau damping. The analysis results in a modified stability diagram which will be used together with physical arguments to explain experimental observations in the Fermilab antiproton accumulator.

Introduction

When residual gas molecules are ionised by Coulomb collisions with beam particles, ions are created with little kinetic energy and in a negatively charged beam may be trapped in the beam. In storage rings, since the beam stays in the machine for very long times, the effects of beam-ion interaction sometimes seriously limit machine performance, see [1] and reference therein.

With the most commonly used clearing electrode systems, the neutralisation level, defined as the ratio of total trapped ion charge to the total beam charge, can be reduced to e.g. a few percent. Still the beam can develop coherent oscillations. Transverse beam coherent oscillations are routinely observed in the Fermilab Antiproton Accumulator, starting with 10-20 mA of stored beam. Oscillations occur near the $(1-q)$ and $(2-q)$ betatron sideband frequencies and also at the $(3-q)$ frequency with larger beam currents. These oscillations can exist at a stable amplitude for a very long time without disastrous effects, however when the beam is cooled longitudinally, as is required before antiproton transfer from the accumulator to the Main Ring, the transverse emittances experience semi-periodic sudden increases without beam loss.

These instabilities are believed to be either caused or at least worsened by the trapped ions. Theories have been developed to deal with two-stream instability[2][3]. The frequency range of these modes is consistent with the spectrum of trapped ion bounce frequencies, but this is also where the resistive wall impedance is the largest. It's very likely that both of them will act at the same time, therefore it is necessary for the theory to include both of them simultaneously. We present a treatment modeled after that in [4] and [5], and use it to explain some of the experimental observations in the Fermilab accumulator.

Theory and Ion Impedance

We only consider transverse dipole oscillations here. To include the effects of non-linear focusing we need to make certain approximations. In general with the presence of nonlinearity, the particle motion is no longer a harmonic oscillation and the focusing is a function of particle's position. We will treat the nonlinearity as simply a modulation of the oscillation frequency as a function of their amplitudes only while maintaining the form of harmonic oscillation. This is a good approximation when the nonlinearity is small. When the nonlinearity is not so small, it can be viewed as keeping only the dipole component and ignoring higher order modes of motion, on a time scale much larger than the oscillation period. In this approximation a particle's motion is described by $y = A\cos(\omega_\beta(A,p)t + \phi_0)$

The Vlasov equations for the beam and trapped ions are

$$\begin{aligned} \frac{\partial \Psi_b}{\partial t} + \dot{y}_b \frac{\partial \Psi_b}{\partial y_b} + F_b \frac{\partial \Psi_b}{\partial \dot{y}_b} + \omega_r \frac{\partial \Psi_b}{\partial \theta} &= 0 \\ \frac{\partial \Psi_i}{\partial t} + \dot{y}_i \frac{\partial \Psi_i}{\partial y_i} + F_i \frac{\partial \Psi_i}{\partial \dot{y}_i} &= 0 \end{aligned}$$

We will ignore the dependence on the asimuthal angle and take an averaged view of all the distributions and forces for simplicity.

The normalised forces are

$$\begin{aligned} F_b &= -\omega_\beta^2 y_b - \omega_{bi}^2 \cdot (y_b - \overline{y_i}) + F_w \\ F_i &= -\omega_{ib}^2 \cdot (y_i - \overline{y_b}) \end{aligned}$$

where ω_β depends individual particle's momentum, ω_{bi} is the angular bounce frequency of the beam in the field of trapped ions, and ω_{ib} that of the ions in the field of beam. All of them may be functions of oscillation amplitudes. F_w is the wake force generated by the normal machine impedance. We have neglected the self-force of both the beam and trapped ions, assuming neutralisation levels are low.

We define action-angle coordinates as

$$\begin{aligned} y_{b,i} &= A_{b,i}\cos\phi_{b,i} \\ \dot{y}_{b,i} &= \omega_{b,i} A_{b,i} \sin\phi_{b,i} \end{aligned}$$

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where the subscript b and i are for beam and trapped ions respectively. The phase space area element, on average, is $dy_{b,i}\,d\dot{y}_{b,i} = \frac{1}{2}d(\omega_{b,i}A^2)\,d\phi_{b,i}$. For simplicity we assume the dependency of the density function Ψ_b on momentum can be separated out, i.e. $\Psi_b(A_b,\phi_b,p) = \Psi(A_b,\phi_b)\,g(p)$.

Following the normal procedure of separating the density function into an stationary part, for which when there is no coherent oscillation, and a small perturbation,

$$\Psi_b = \Psi_{b0} + \Psi_{b1}e^{i(n\theta-\Omega t)}$$
$$\Psi_i = \Psi_{i0} + \Psi_{i1}e^{i(n\theta-\Omega t)}$$

The coherent oscillations $\overline{y}_{b,i} = Y_{b,i}\,e^{i(n\theta-\Omega t)}$ have amplitudes of $Y_{b,i} \equiv \frac{1}{2\pi}\int \Psi_{b,i1}\,A_{b,i}\cos\phi_{b,i}\,d(\omega_{b,i}A^2_{b,i})\,d\phi_{b,i}$. The wake force is

$$F_w = i\frac{Ne^2}{\gamma m_b T_0 L}Z(\Omega)Y_b e^{i(n\theta-\Omega t)}$$

where $Z(\Omega)$ is the machine impedance[5].

The linearised Vlasov equations

$$-i(\Omega - n\omega_r)\Psi_{b1} - \omega_b\frac{\partial\Psi_{b1}}{\partial\phi_b} + \left(\frac{iNe^2}{\gamma m_b T_0 L Z(\Omega)}Y_b + \omega^2_{bi}Y_i\right)\frac{\partial\Psi_{b0}}{\partial\dot{y}_b}=0$$

$$-i\Omega\Psi_{i1} - \omega_i\frac{\partial\Psi_{i1}}{\partial\phi_i} + \omega^2_{ib}Y_b\frac{\partial\Psi_{i0}}{\partial\dot{y}_i}=0$$

lead to the following dispersion equation

$$1 = i\frac{Ne^2 Z(\Omega)}{\gamma m_b T_0 L}\cdot\int\frac{\Psi'_{b0}(A_b)\omega_b A^2_b\,dA_b\,g(p)dp}{(\Omega-n\omega_r)^2-\omega^2_b} + \int\frac{\Psi'_{b0}(A_b)\omega^2_{bi}\omega_b A^2_b\,dA_b\,g(p)dp}{(\Omega-n\omega_r)^2-\omega^2_b}\int\frac{\Psi'_{i0}(A_i)\omega^2_{ib}\omega_i A^2_i\,dA_i}{\Omega^2-\omega^2_i}$$

In general ω^2_{bi} is a function of A_b and therefore cannot be pulled out of the integration, but if it is only a weak function of A_b, or if ω_r and ω_b are not dependent on A_b, which is almost true, then ω^2_{bi}, or another quantity of the same order, can be taken out as a constant. This constant, let us call it ω^2_c, measures the average coupling strength from ions to the beam. In the limit that ω_r and ω_b do not depend on A_b ω_c is roughly the rms bounce frequency of beam in the field of trapped ions. In this approximation the dispersion equation becomes

$$1 = i\frac{Ne^2}{\gamma m_b T_0 L}\int\frac{g(p)\,dp}{\omega^2_b-(\Omega-n\omega_r)^2}\cdot[Z(\Omega)+Z_i(\Omega)]$$

where

$$Z_i(\Omega) = -i\frac{\gamma m_b T_0 L\omega^2_c}{Ne^2}\cdot\int\frac{\Psi'_{i0}(A_i)\omega^2_{ib}\omega_i A^2_i\,dA_i}{\Omega^2-\omega^2_i}$$

As we can see, the main effect of trapped ions comes in the form of extra impedance, denoted as "ion impedance", Z_i. The other contribution of ions is to shift the beam oscillation frequency, which is contained in the term $\omega^2_b = \omega^2_\beta + \omega^2_{bi}$. This shift, however, is usually tiny and can be ignored.

It is, however, very difficult to know the details about this ion impedance Z_i, because it depends on the neutralisation profile, ion amplitude distribution, etc. None of these are known in detail. Instead, we will try to get a rough estimation on Z_i and determine a qualitative picture of the effect. For this purpose we assume trapped ions have the same transverse distribution as the beam, and take this distribution as a round Gaussian with an rms sise that is the machine average.

The coefficient $\gamma m_b T_0 L\omega^2_c/Ne^2$ in Z_i is independent of the beam intensity and proportional to the machine neutralisation level, e.g. for the Gaussian transverse distribution

$$\omega^2_c = \frac{\eta Ne^2}{L\gamma m_b}\frac{1}{\sigma_y(\sigma_x+\sigma_y)}$$

and at the stability boundary the ion impedance is

$$Z_i = \frac{\pi T_0}{\sigma_y(\sigma_x+\sigma_y)}\left\{\frac{\pi}{2}\Psi'_{i0}(\Omega)\Omega^2A^2_i(\Omega) - i\frac{\Omega}{2}P.V.\int\frac{\Psi'_{i0}(\omega_i)\omega_i A^2_i(\omega_i)\,d\omega_i}{\omega_i-\Omega}\right\}$$

This ion-impedance will tend to damp either the slow wave or fast wave, and anti-damp the other, depending on the ion distribution. Usually, more ions are distributed in the small amplitude, high oscillation frequency region, and the distribution tapers off toward large amplitudes, and consequently we usually have a positive real "ion resistance". This will damp the fast wave and anti-damp the slow wave which is also anti-damped by the normal machine impedance, i.e. the "ion impedance" will reduce the machine impedance budget and may cause beam instability in cases that are otherwise stable. For illustration let us take the simplified picture described above and evaluate some typical numbers from the Fermilab accumulator and plot the stability diagrams with 0% and 1% neutralisation. It can be seen from Fig. 1 that it does not take a high level of ion neutralisation to destabilise an otherwise stable situation, and it is possible to have no stable region at all.

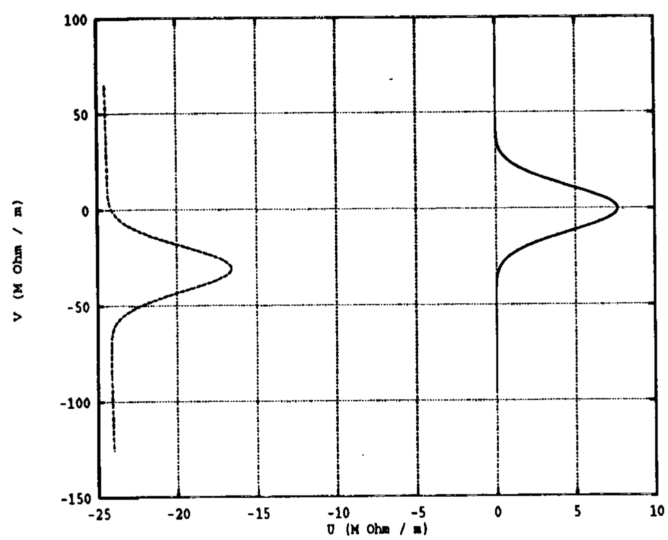

Figure 1: Beam Stability Diagrams with 0 and 1% Neutralization

We have to point out two points. First, the theory unavoidably overestimates the effect of ions because of the neglect of the longitudinal motion of ions. This motion causes resonant ions to lose the phase information needed to stay resonant. Secondly, this theory is only a perturbative treatment and not a self consistent one. We started with an equilibrium distribution and evaluated the stability of any small perturbation, but the unstable motions may change the "equilibrium" distribution, especially trapped ions'. To develop a fully self-consistent theory is very difficult. However, we can use an iterative process where the theory together with some physical arguments form a closed loop and we can at least get a qualitative explanation of what we have observed, which we will carry out in the following section.

Experimental Observations

In the Fermilab Antiproton Accumulator, ion trapping has been, at least partly, a limit to the normal operation. To correct the problem an ion clearing electrode system has been installed in the accumulator and it has recently been upgraded from the previous fixed $100V$ to a variable one with up to $1KV$ voltage.

Experiments have shown that ion trapping contributes to beam coherent oscillations. Fig. 2 shows the beam coherent oscillation strengths with different amount of neutralisation achieved by varying clearing voltage in one sixth of the machine at $900, 100, 50$ and 10 volts, where power in all three coherent lines grows with decreasing clearing voltage. Monitoring the coherent line power with time shows that they can change rather abruptly. Fig. 3 shows one example.

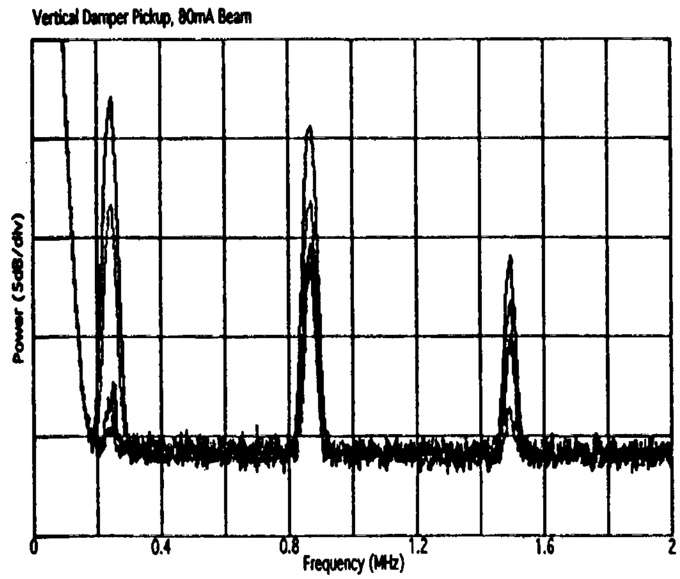

Figure 2: Coherent Power Spectrum with Various Clearing Voltages

The above can be explained with the model we presented in the previous section, that as beam current increases the ion impedance will gradually push the otherwise stable beam into the unstable region and cause the beam-ion coherent oscillation to grow. Unlike the normal machine impedance, the ion impedance changes as the beam-ion interaction alters the "equilibrium" ion distribution. When the beam develops an instability, resonant ions also undergo oscillation growth. Since the beam is normally much stronger and more rigid, the initial growth in the coherent oscillation tends to heat the ion distribution and drive the resonant ions to larger amplitudes. As a result, the ion distribution will be distorted so that the beam becomes semi-stable, and the coherent oscillation strength will be reduced, just as shown in Fig. 3. The beam coherent oscillation is needed to maintain this modified ion distribution as the creation of new ions and the natural tendency of ions without heating will bring back the original ion distribution which made the beam to grow in the first place. This process will be enhanced through the increase of beam current and/or the reduction of the beam momentum spread and the increase of ion trapping, where either the beam moves closer to instability intrinsically because of the weakening of Landau damping or trapped ions become stronger or both can occur. When the trapped ions are strong enough and the instability growth is too fast for ions to respond a true instability will develop and beam transverse emittances will suffer explosive growth as has been observed.

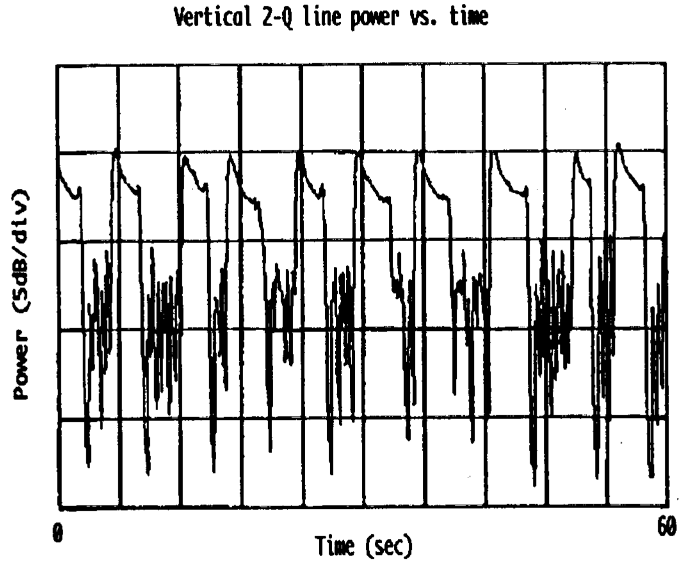

Figure 3: Coherent Oscillation Power Change with Time

Acknowledgment

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References

[1] John Marriner, *et al.* Experiments and practise in beam shaking. *Particle Accelerators*, 30:13–20, 1990.

[2] D.G. Koshkarev and P.R. Zenkevich. Resonance of coupled transverse oscillations in two circular beams. *Particle Accelerators*, 3:1–9, 1972.

[3] L.J. Laslett, A.M. Sessler and D. Möhl. Transverse two-stream instability in the presence of strong species-species and image forces. *Nucl. Instr. Meth.*, 121:517–524, 1974.

[4] L. Jackson Laslett, V. Kelvin Neil, and Andrew M. Sessler. Transverse resistive instabilities of intense costing beams in particle accelerators. *Rev. Sci. Inst.*, 36(4):436, 1965.

[5] A.W. Chao. Coherent instabilities of a relativistic bunched beam. *SLAC Internal Note*, SLAC-PUB-2946, June 1982.